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L2 – Hydraulic Energy Conversion

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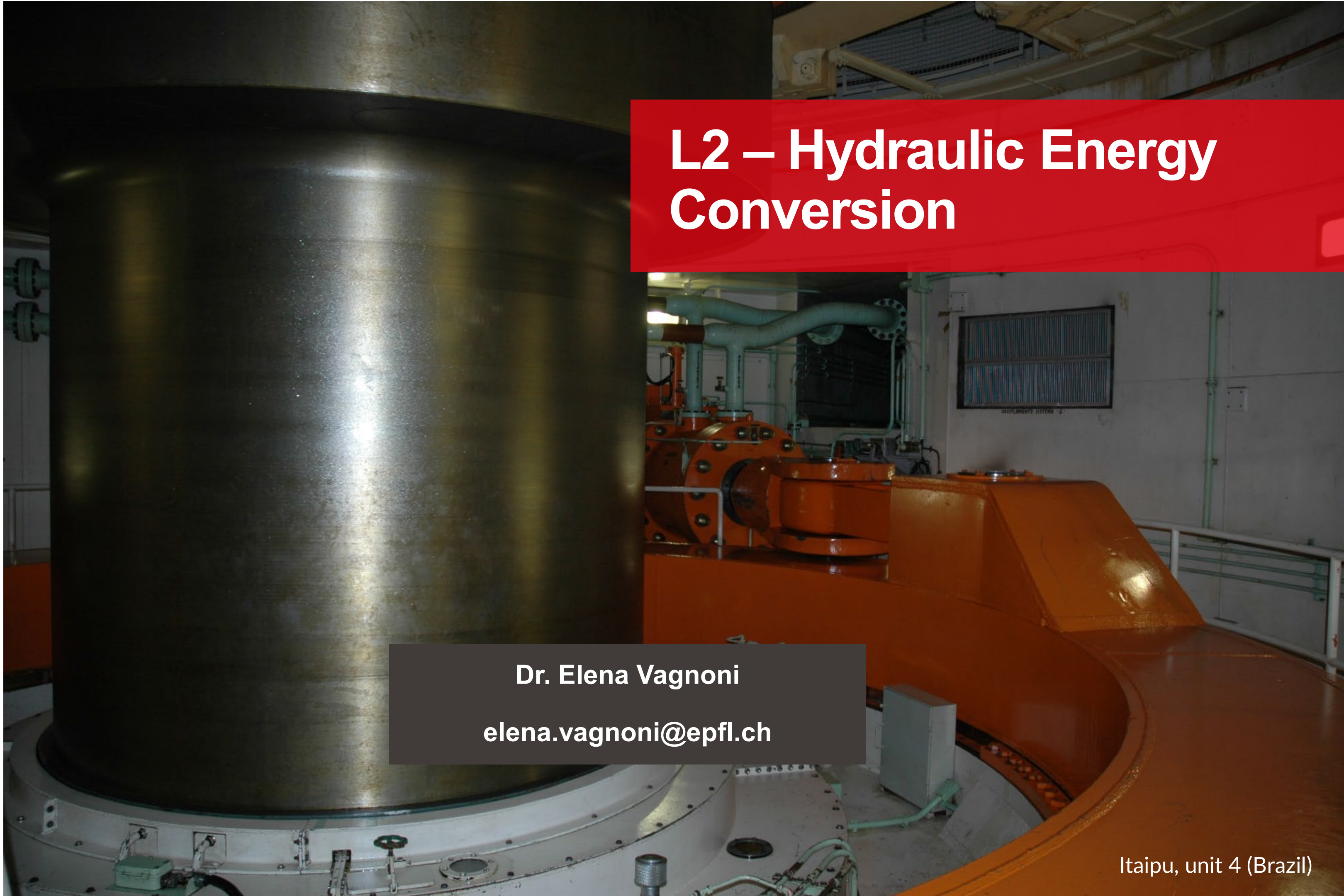


Itaipu, unit 4 (Brazil)

École polytechnique fédérale de Lausanne

Topics of the lecture

- Power Balance
- Losses & Efficiency
- Machine Characteristic curves

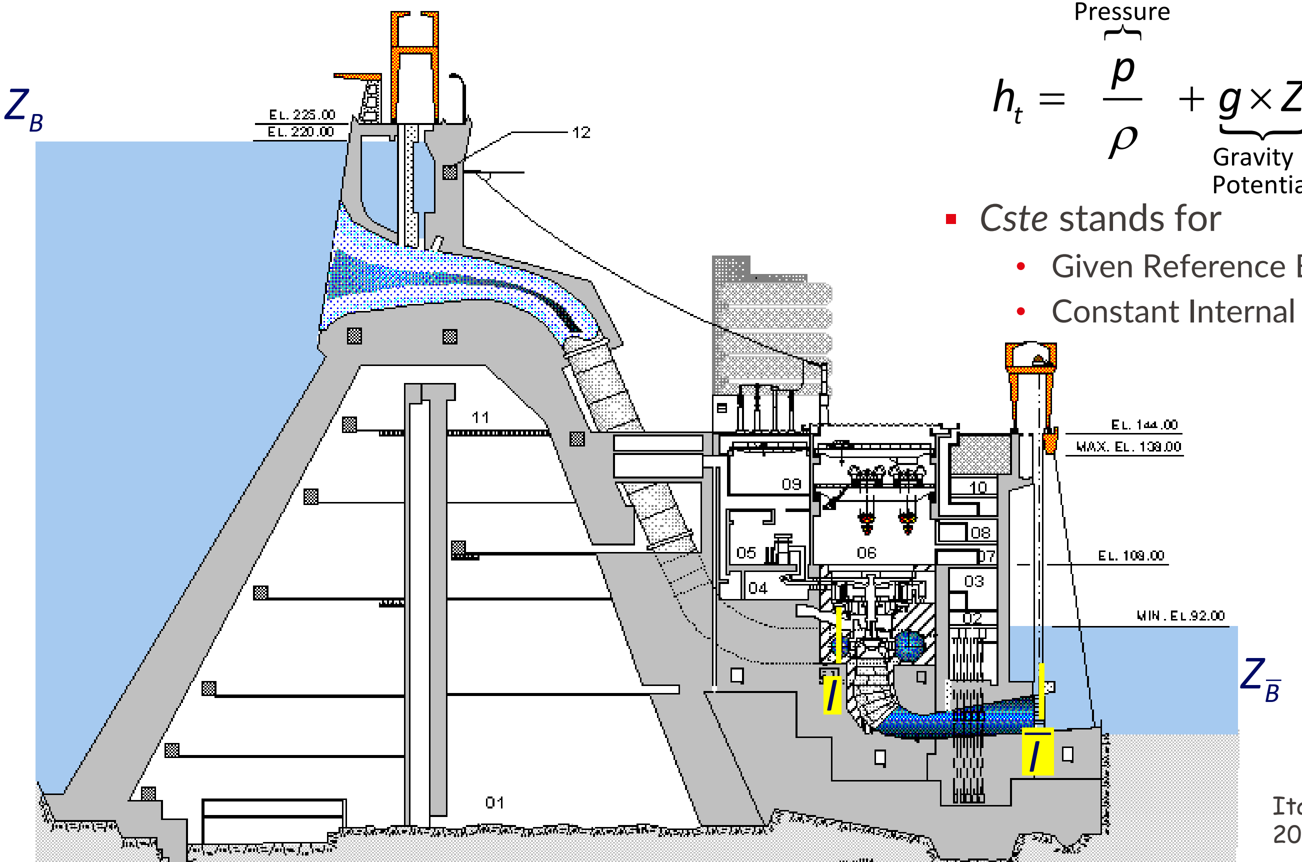
From L1: Specific Hydraulic Energy

Specific Energy Balance

- Local Mean Flow Specific Energy: Flow Property

$$h_t = \overbrace{\frac{p}{\rho}}^{\text{Pressure}} + \underbrace{g \times Z}_{\text{Gravity Potential}} + \overbrace{\frac{\vec{C}^2}{2}}^{\textit{Kinetic}} + Cste \quad \left(\mathrm{J \cdot kg^{-1}}\right)$$

- *Cste* stands for
  - Given Reference Elevation Z_{ref} (e.g. Sea Level)
  - Constant Internal Specific Energy



Available Specific Hydraulic Energy

$$\boxed{\begin{aligned} E &\triangleq gH_I - gH_{\bar{I}} \\ &= g\left(Z_B - Z_{\bar{B}}\right) - \sum gH_r^T \end{aligned}}$$

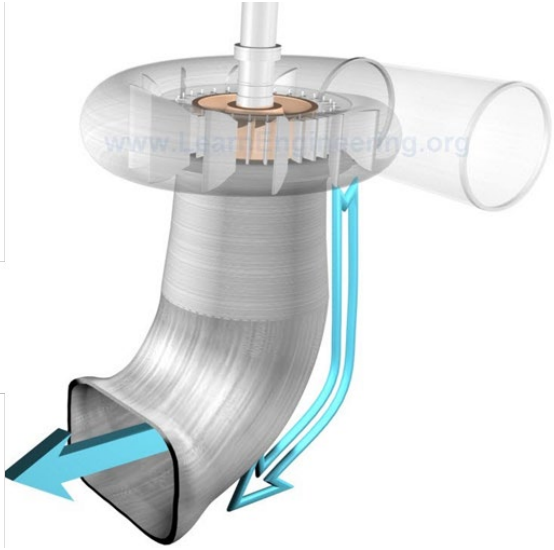
Itaipú, Brazil/Paraguay
20 x 740 MW Francis

From L1: Turbomachines definition

- Turbomachines are machines including stator and rotor elements that transfer energy between a rotor and a fluid.
- Energy transfer consists in the conversion between hydraulic energy and mechanical work

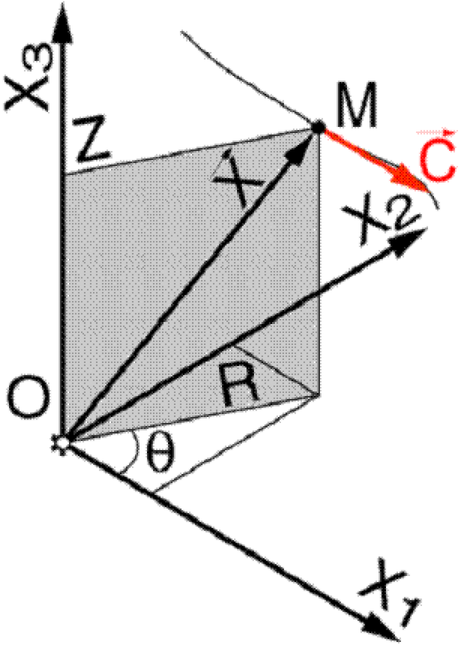
Remember! Mass and energy transfer are evaluated through:

- Mass balance equation
- First thermodynamic principle
- Second thermodynamic principle



Turbomachines definition

Frame of reference



- Frame of Reference
  - Cartesian $\{O;X_i\}$
  - Cylindrical $\{O;R,\theta,Z\}$
- Absolute Flow Field
  - Velocity $\vec{C}$
  - Acceleration

$$\frac{D\vec{C}}{Dt}=\frac{\partial\vec{C}}{\partial t}+\left(\vec{C}\cdot\vec{\nabla}\right)\vec{C}$$
$$=\frac{\partial\vec{C}}{\partial t}+\vec{\nabla}\frac{\vec{C}^2}{2}-\vec{C}\times\vec{\Omega}$$

Hydraulic machine definitions

Flow Driving Equation

- Incompressible Flow $\frac{1}{\rho}\frac{D\rho}{Dt} = -\vec{\nabla}\cdot\vec{C} = 0$

- Viscous & Turbulent Flow
  - Reynolds Averaged Navier-Stokes Equation

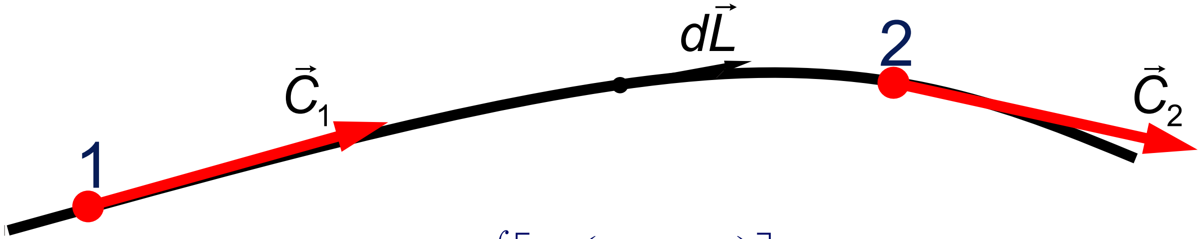
$$\frac{D\vec{C}}{Dt} = \frac{\partial\vec{C}}{\partial t} + \vec{\nabla}\frac{\vec{C}^2}{2} - \vec{C}\times\vec{\Omega}$$

$$= -\vec{\nabla}\left(\frac{\overbrace{p}^{\text{Pressure}}}{\underbrace{\rho}_{\text{Density}}} + \overbrace{gZ}^{\text{Potential}}\right) + \vec{\nabla}\cdot\left(\underbrace{2\nu\overline{\overline{D}}}_{\text{Visc. Stress}} + \underbrace{\frac{\overline{\overline{\tau_t}}}{\rho}}_{\text{Turb. Stress}}\right)$$

Specific Hydraulic Energy

Specific Energy Balance

- Local Balance Along a Streamline

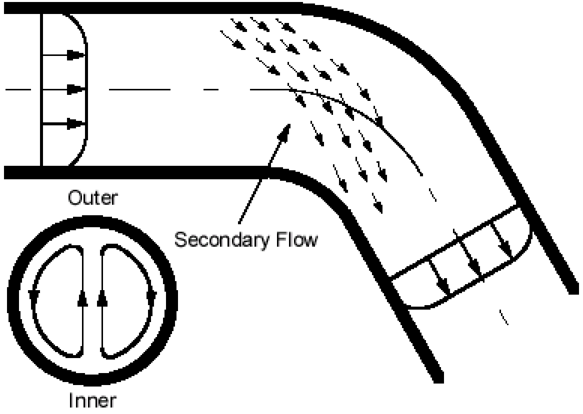


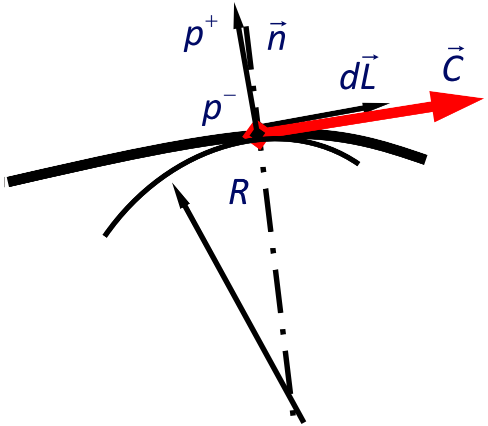
$$h_1 = h_2 - \int_{\overset{\frown}{12}} \left[\vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right) \right] \cdot d\vec{L} + \int_{\overset{\frown}{12}} \frac{\partial \vec{C}}{\partial t} \cdot d\vec{L}$$

Specific Hydraulic Energy

Specific Energy Balance

$$\frac{d}{dn}\left(\frac{p}{\rho}+gZ\right)=-\frac{C^2}{R}+\vec{\nabla}\cdot\left(2\nu\overline{\overline{D}}+\frac{\overline{\overline{\tau}}_t}{\rho}\right)\cdot\vec{n}$$





- For straight flows !

$$R\rightarrow\infty\Rightarrow\frac{d}{dn}\left(\frac{p}{\rho}+gZ\right)=0$$

Specific Hydraulic Energy

Mean Flow and Power balance

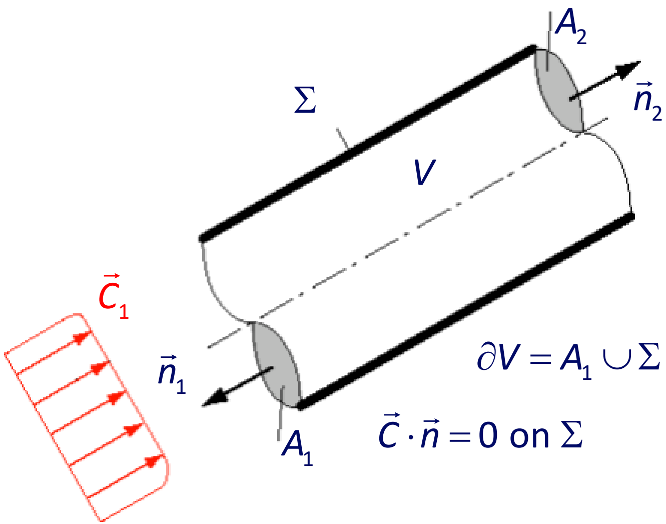
- Flow Power Balance for a Pipe Domain V

$$\int_{A_1\cup A_2} \rho h_t \vec{C}\cdot\vec{n}dA = -\int_V \overbrace{\frac{\partial}{\partial t}\left(\frac{\vec{C}^2}{2}\right)}^{\equiv 0 \text{ if Stationary}} \rho dV + \overbrace{\int_{A_1\cup A_2}\left(\rho\left(2\nu\overline{\overline{D}}+\frac{\overline{\overline{\tau}}_t}{\rho}\right)\cdot\vec{C}\right)\cdot\vec{n}dA}^{\equiv 0 \text{ if Homogeneous}}$$

$$-\int_V (\Phi+\Pi)\rho dV \quad (\mathrm{W})$$

- Flow Power Dissipation

$$P_{r1\div 2} = -\int_V \left(\underbrace{\overbrace{2\nu\,\overline{\overline{D}}^2}^{\Phi=\text{Viscosity}} + \overbrace{\frac{\overline{\overline{\tau}}_t}{\rho}:\overline{\overline{D}}}^{\Pi=\text{Turbulence}}}_{\text{Specific Power }(\mathrm{W\cdot kg^{-1}})}\right)\rho dV$$



$$\partial V = A_1 \cup \Sigma \cup A_2$$

$$\vec{C}\cdot\vec{n} = 0 \text{ on } \Sigma$$

Specific Hydraulic Energy

Mean flow power balance

- Flow Weighted Average of Local Specific Total Enthalpy

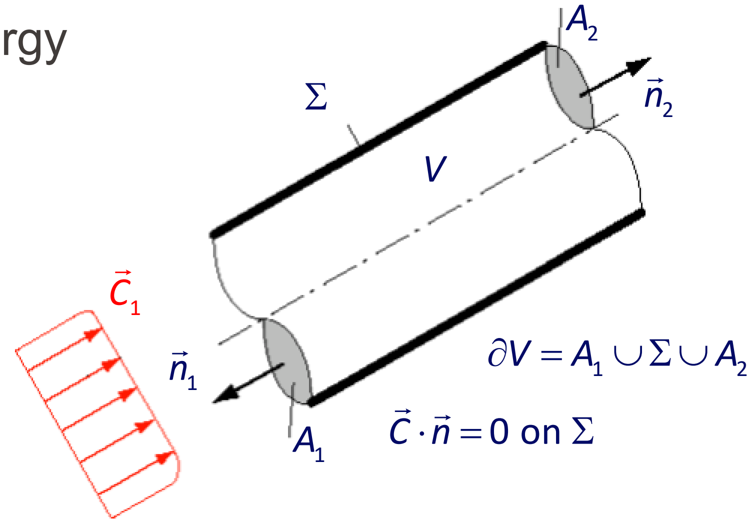
$$gH \triangleq \frac{P_h}{\rho Q} = \pm\int_A \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2}\right)\frac{\vec{C}\cdot\vec{n}dA}{Q} \geq 0 \qquad \left(\mathrm{J\cdot kg^{-1}}\right)$$

- Balance of Mean Specific Energy

$$gH_1 = gH_2 + \frac{1}{Q}\int_V (\Phi + \Pi)dV$$

$$-\frac{1}{Q}\overbrace{\int_{A_1\cup A_2}\left(\left(2\nu\overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho}\right)\cdot\vec{C}\right)\cdot\vec{n}dA}^{\equiv 0 \text{ if Homogeneous}}$$

$$+\frac{1}{Q}\int_V \overbrace{\frac{\partial}{\partial t}\left(\frac{\vec{C}^2}{2}\right)}^{\equiv 0 \text{ if Stationary}} dV$$



Specific Hydraulic Energy

Flow power dissipation

- Viscous Dissipation
  Production of Turbulence

$$P_{r1\div 2} = -\int_V \left(\underbrace{\overbrace{2\nu \,\overline{\overline{D}}^{2}}^{\text{Viscosity}} + \overbrace{\frac{\overline{\overline{\tau}}_t}{\rho}}^{\text{Turbulence}} : \overline{\overline{D}}}_{\text{Specific Power } \left(\text{W}\cdot\text{kg}^{-1}\right)} \right) \rho dV$$

$$+ \int_{A_1 \cup A_2} \rho \left[\left(2\nu \,\overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right) \cdot \vec{C} \right] \cdot \vec{n} dA - \int_V \frac{\partial}{\partial t} \left(\frac{C^2}{2} \right) \rho dV$$

Specific Hydraulic Energy

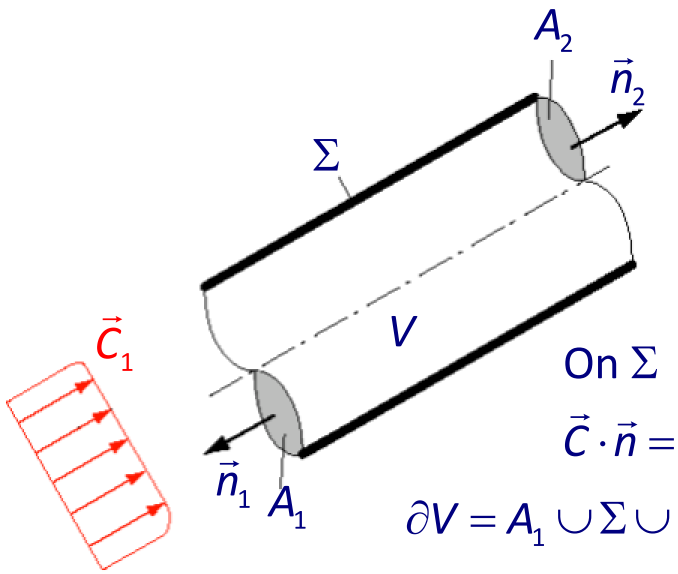
Discharge and power definition

- Discharge

$$Q = \int_A \vec{C} \cdot \vec{n} dA \quad \left(\mathrm{m}^3 \cdot \mathrm{s}^{-1}\right)$$

- Flow Power

$$P_h = \int_A \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} dA \quad (\mathrm{W})$$



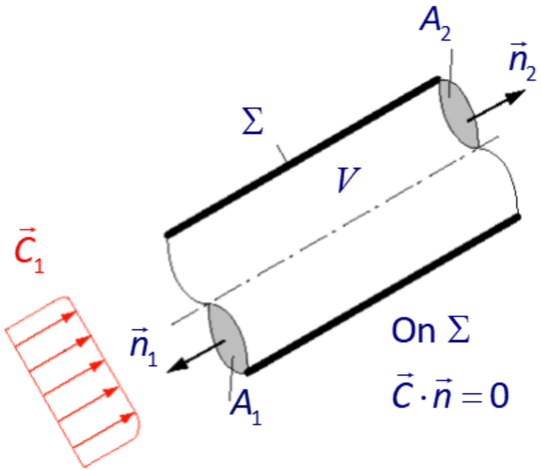
On Σ

$\vec{C} \cdot \vec{n} = 0$

$\partial V = A_1 \cup \Sigma \cup A_2$

Specific Hydraulic Energy

Flow and Power balance



$$\partial V = A_1 \cup \Sigma \cup A_2$$

- Discharge $Q_1 = Q_2$
- Hydraulic Power $P_{h1} = P_{h2} + P_{r1 \div 2}$

- Specific Hydraulic Energy

$$gH = \frac{P_h}{\rho Q}$$

$$gH_1 = gH_2 + gH_{r1 \div 2}$$

$$gH \triangleq \frac{P_h}{\rho Q} = \int_A \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \frac{\vec{C} \cdot \vec{n}}{Q} dA \geq 0 \quad \left(\mathrm{J \cdot kg^{-1}} \right)$$

Specific Hydraulic Energy

Hydraulic System: Specific Energy and Discharge Balance

- Specific Energy Balance $gH_1 = gH_2 + \sum_{1 \div 2} \left(gH_r\right)_i$
  - Steady Flow
  - Straight Stream Lines, free of Secondary Flow

- Discharge Balance $Q_1 = Q_2$
  - Mass conservation $A_1 \times C_1 = A_2 \times C_2$

Specific Hydraulic Energy

Specific Energy Balance

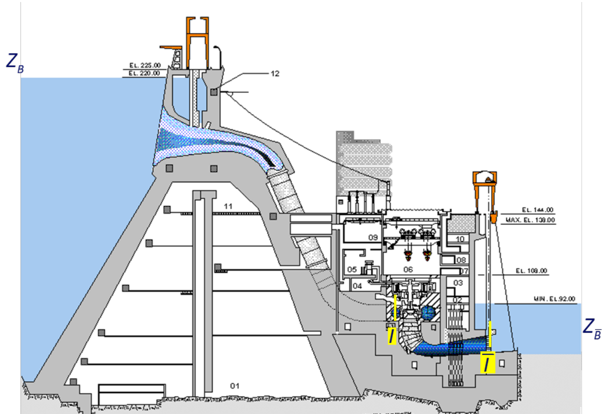
- Headwater - High Energy Side $$gH_B = \frac{p_{atm}}{\rho} + gZ_B + 0 = gH_I + \sum_{\text{Headwater Side}} gH_r$$

- Low Energy Side - Tailwater Side $$gH_{\bar{I}} = gH_{\bar{B}} + \sum_{\text{Tailwater Side}} gH_r$$

$$gH_{\bar{B}} = \frac{p_{atm}}{\rho} + gZ_{\bar{B}} + 0$$

- Available Specific Energy for the Turbine

$$E = gH = gH_I - gH_{\bar{I}} = g \times \left(Z_B - Z_{\bar{B}} \right) - \sum gH_r$$



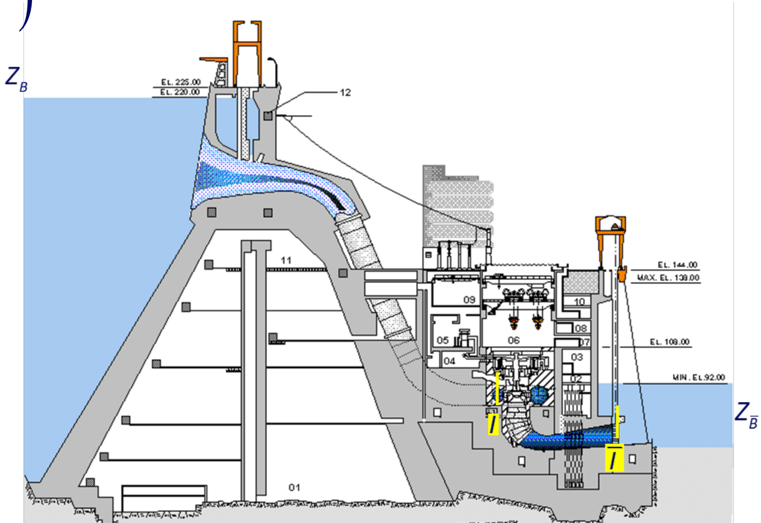
Specific Hydraulic Energy

Hydraulic machine: Specific Energy balance

- Machine Flow Boundaries
  - High Energy Side A_I
  - Low Energy Side $A_{\bar{I}}$
- Specific Hydraulic Energy $E \doteq gH_I - gH_{\bar{I}} \quad \left(\mathrm{J \cdot kg^{-1}}\right)$
  - Always Positive Value $E \doteq gH_I - gH_{\bar{I}} \geq 0$
  - Available Specific Energy for the Turbine

$$E = gH \quad \left(\mathrm{J \cdot kg^{-1}}\right)$$

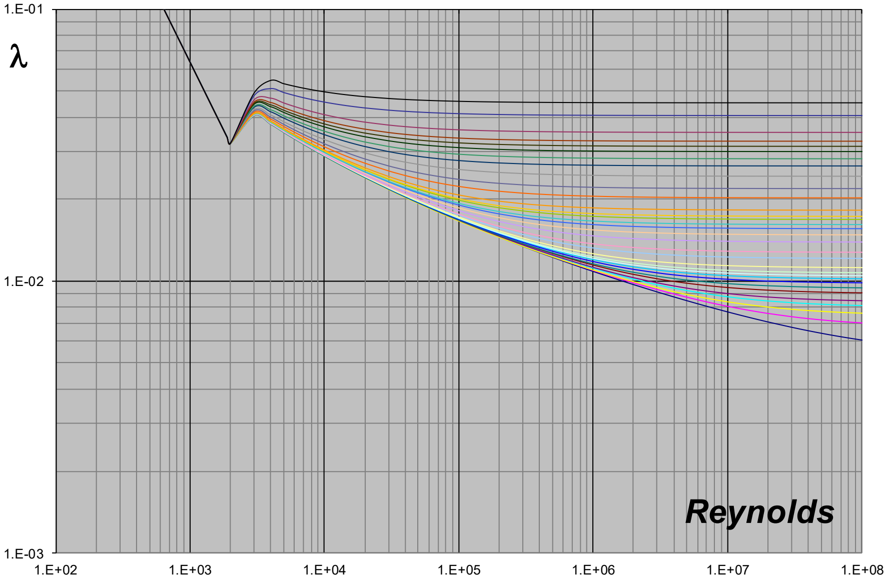
- Net Head $H = \dfrac{E}{g} \quad (\mathrm{mW.C}).$



Specific Hydraulic Energy

Specific Energy Losses : The Pipe Distributed Losses

$$gH_{r1\div2} = \lambda\left(Re;\frac{k}{D_h}\right)\frac{L_{\overset{\frown}{12}}}{D_h}\frac{\vec{C}^2}{2} \quad \left(\mathrm{J\cdot kg^{-1}}\right)$$



- λ Local Loss Coefficient $\lambda = \lambda\left(\mathrm{Re}, \frac{k}{D_h}\right)$
- Re Reynolds Number $\mathrm{Re} = \frac{CD_h}{\nu}$
- k Roughness
- D_h Hydraulic Diameter $D_h = \frac{4A}{P_{wet}}$

Specific Hydraulic Energy

Specific Energy Losses : The Pipe Distributed Losses

Churchill Empirical Formula

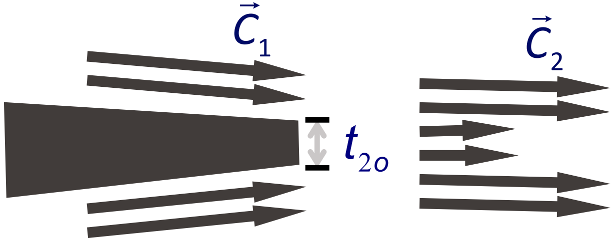
$$\lambda\left(\mathrm{Re};\frac{k_s}{D}\right)=8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12}+\frac{1}{\left(A+B\right)^{\frac{3}{2}}}\right]^{\frac{1}{12}}$$

$$\text{with}\quad A=\left[2.457\cdot\ln\frac{1}{\left(\frac{7}{\mathrm{Re}}\right)^{0.9}+0.27\cdot\frac{k_s}{D}}\right]^{16}$$

$$\text{and}\quad B=\left[\frac{37'530}{\mathrm{Re}}\right]^{16}$$

Specific Hydraulic Energy

Specific Energy Losses : Singular Losses



- Turbulent Mixing
  - Sudden Expansion
  - Wake
  - Flow Separation
  - etc...
- For the full list and their calculation refer to pages 35-38 of the hand-out on Moodle ☺

$$gH_{r1\div 2} \approx \left(1-\frac{A_1}{A_2}\right)^2 \frac{C_1^2}{2} = \left(1-\frac{A_2}{A_1}\right)^2 \frac{C_2^2}{2} = \left(\frac{\delta A}{A_2-\delta A}\right)^2 \frac{C_2^2}{2}$$

Specific Hydraulic Energy

Specific Energy Losses : Total Losses

- Distributed Losses : $gH_{r1\div2} = \lambda\left(Re;\frac{k}{D_h}\right)\frac{16}{\pi^2}\frac{L_{\widehat{12}}}{D_h^5}\frac{Q^2}{2} \quad \left(\mathrm{J\cdot kg^{-1}}\right)$

- Singular Losses for the ith Component $gH_{ri} = K_i\frac{1}{A_i^2}\frac{Q^2}{2} \quad \left(\mathrm{J\cdot kg^{-1}}\right)$

- General Formula $gH_{rx\div y} = \sum_{i=1}^{n} K_i\frac{1}{A_i^2}\times\frac{Q^2}{2} \quad \left(\mathrm{J\cdot kg^{-1}}\right)$

$$= \underbrace{\sum_{i=1}^{n} K_i\frac{A_{ref}^2}{A_i^2}}_{K_{Inst.}}\times\frac{Q^2}{2A_{ref}^2} = K_{Inst.}\times\frac{Q^2}{2A_{ref}^2}$$

Example

Itaipu (Brasil) energy losses

- Compute the potential specific energy by knowing:

  $Z_B = 220\ \text{m};\ Z_{\overline{B}} = 99\ \text{m}$

  $g_{@\ \text{Itaipu}} = 9.79\ \text{m}\cdot\text{s}^{-2}$

- Assuming 2 % losses, compute available Specific Energy:
  And the net Head:
- Compute the available Specific Energy for a 5 % discharge decrease:

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Example

Itaipu (Brasil) energy losses

- Potential specific energy: $g(Z_B - Z_{\overline{B}}) = 1'184.6\ \mathrm{J \cdot kg^{-1}}$

  $Z_B = 220\ \mathrm{m};\ Z_{\overline{B}} = 99\ \mathrm{m}$

  $g_{@\ \mathrm{Itaipu}} = 9.79\ \mathrm{m \cdot s^{-2}}$

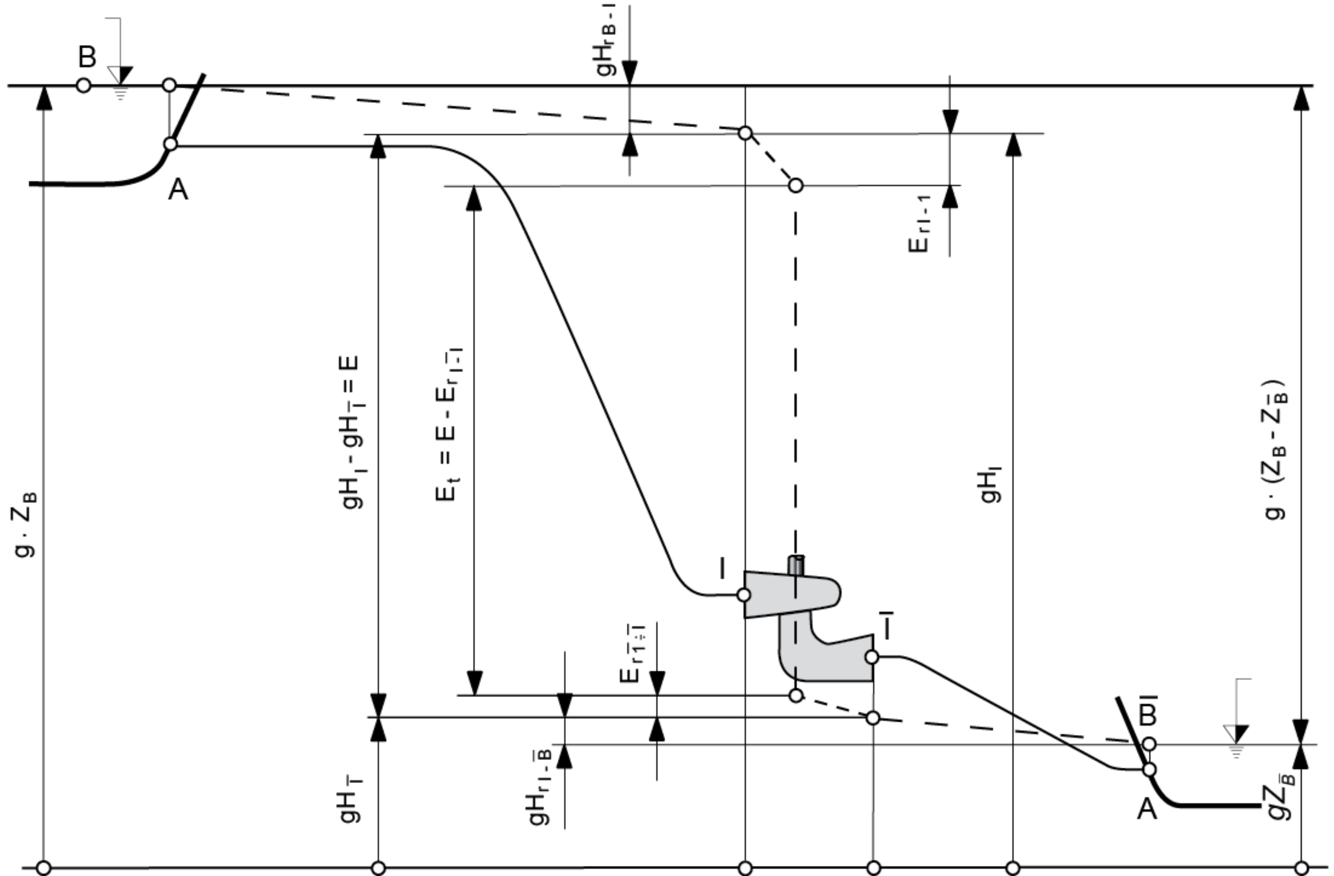
- Assuming 2 % losses
  Available Specific Energy: $gH_I - gH_{\overline{I}} = \left(g \cdot (Z_B - Z_{\overline{B}})\right) \cdot \left(1 - 0.02\right) = 1'160.9\ \mathrm{J \cdot kg^{-1}}$
  "Net Head": $H = 118.6\ \mathrm{m}$

- Available Specific Energy for a 5 % discharge decrease

  $gH_I - gH_{\overline{I}} = \left(g \cdot (Z_B - Z_{\overline{B}})\right) \cdot \left(1 - 0.02 \cdot 0.95^2\right) = 1'163.2\ \mathrm{J \cdot kg^{-1}}$

Specific Hydraulic Energy

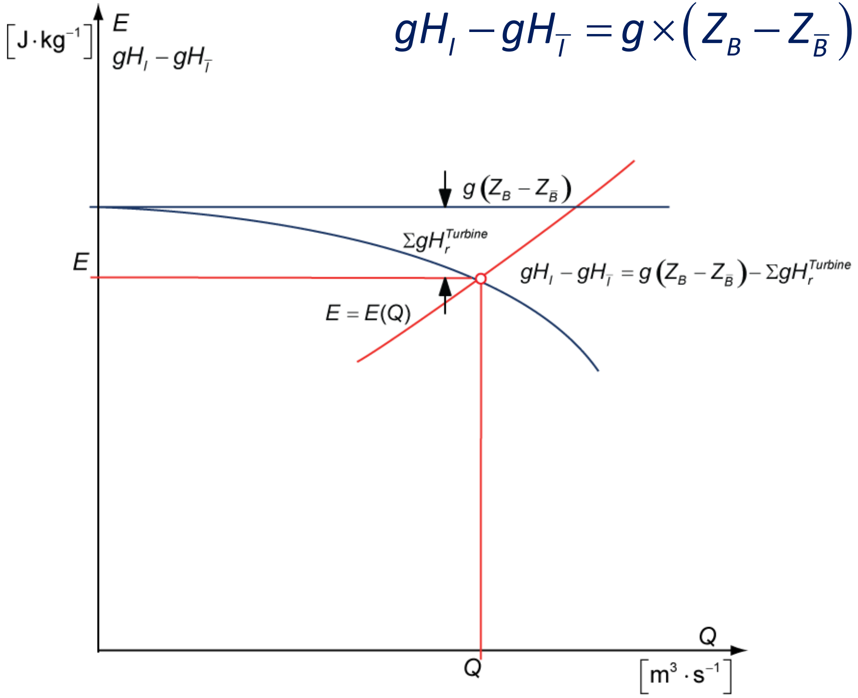
Specific Energy Balance: generating mode



Specific Hydraulic Energy

Hydraulic Characteristics : generating mode

$$gH_I - gH_{\bar{I}} = g \times \left(Z_B - Z_{\bar{B}}\right) - \sum gH_r$$



Specific Hydraulic Energy

Hydraulic Drive : generating mode

- Machine Power Output

$$P = \vec{\omega} \cdot \vec{T} \quad (\mathrm{W})$$

- Available Hydraulic Power

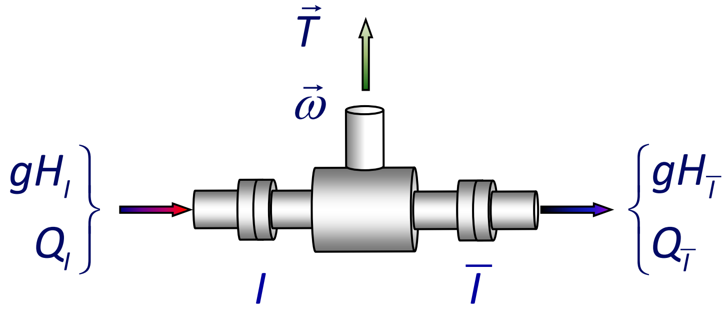
$$\underbrace{P_h}_{\frac{\mathrm{J}}{\mathrm{s}}} = \underbrace{\rho Q}_{\frac{\mathrm{kg}}{\mathrm{m}^3} \times \frac{\mathrm{m}^3}{\mathrm{s}}} \times \underbrace{E}_{\frac{\mathrm{J}}{\mathrm{kg}}} \quad (\mathrm{W})$$

- Turbine Efficiency

$$P = \eta^T \times P_h \quad ; \eta^T \leq 100\ \%$$

- Driving Power Defined as Positive :

$$P \geq 0$$



$$E \triangleq gH_I - gH_{\bar{I}} \quad \left(\mathrm{J} \cdot \mathrm{kg}^{-1}\right)$$

$$P_h = \rho Q \times E$$

$$= \rho Q \times \left(gH_I - gH_{\bar{I}}\right)$$

Specific Hydraulic Energy

Rotating Train Dynamics

- Rotating train angular momentum equation: $J \times \dfrac{d\omega}{dt} = T + T_{el} \quad (\mathrm{N \cdot m})$
  - Inertia: J
  - Angular speed: ω
  - Synchronous conditions : $T = -T_{el}$
  - Load rejection : runaway speed $T_{él.} = 0 \Rightarrow \dfrac{d\omega}{dt} = T > 0$

- Synchronous speed relation :
  - Grid frequency $f_{grid} = 16\tfrac{2}{3}$ Hz; 50 Hz; 60 Hz
  - Number of poles of the synchronous generator z_p
  - Rotating frequency $n = \dfrac{2 \times f_{grid}}{z_p} \quad (\mathrm{Hz})$

Example

Itaipu (Brasil) rotating train characteristics

- For a discharge of Hydraulic Power:
- For 5% less discharge

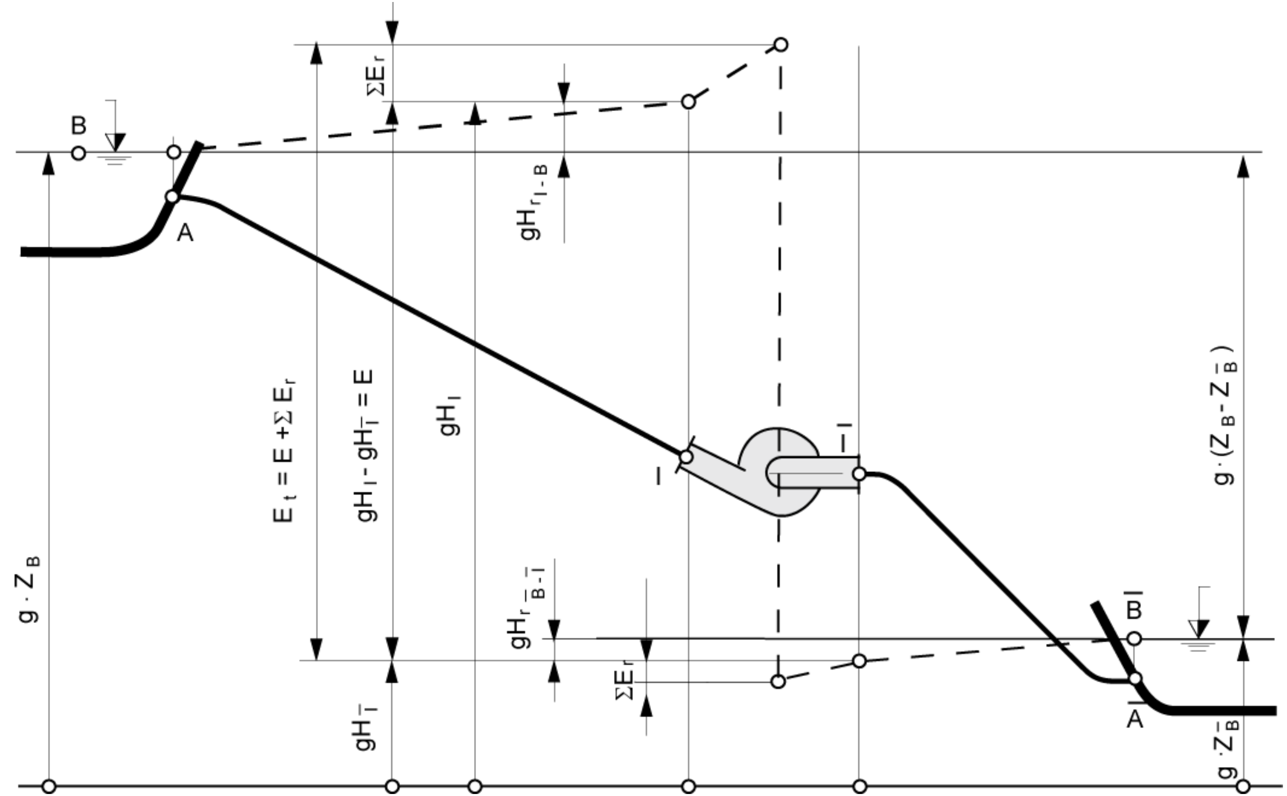
$$Q = 677 \text{ m}^3 \cdot \text{s}^{-1}$$
$$P_h = 786 \text{ MW}$$
$$P_h = 748 \text{ MW}$$

- For the Brazilian units number of poles
- For the Paraguayan units number of poles

$$f_{grid} = 60 \text{ Hz}; N = 92.3 \text{ min}^{-1}$$
$$z_p = 78$$
$$f_{grid} = 50 \text{ Hz}; N = 90.9 \text{ min}^{-1}$$
$$z_p = 66$$

Specific Hydraulic Energy

Specific Energy Balance: pumping mode



Specific Hydraulic Energy

Specific Energy Balance: pumping mode

- Low Energy Side - Tailwater Side $$gH_{\bar{B}} = \frac{p_{atm}}{\rho} + gZ_{\bar{B}} + 0$$

$$gH_{\bar{B}} = gH_{\bar{I}} + \sum_{\text{Tailwater Side}} gH_r$$

- High Energy Side-Headwater $$gH_I = gH_B + \sum_{\text{Headwater Side}} gH_r$$

$$gH_B = \frac{p_{atm}}{\rho} + gZ_B + 0$$

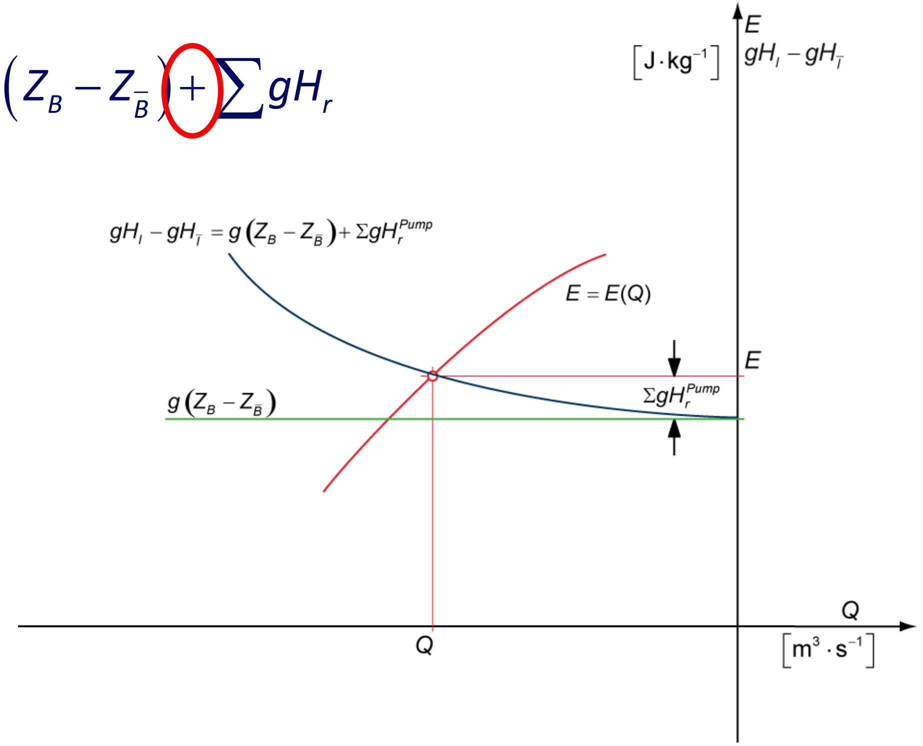
- Specific Energy Supplied by the Pump

$$gH_I - gH_{\bar{I}} = g \times \left(Z_B - Z_{\bar{B}} \right) + \sum gH_r$$

Specific Hydraulic Energy

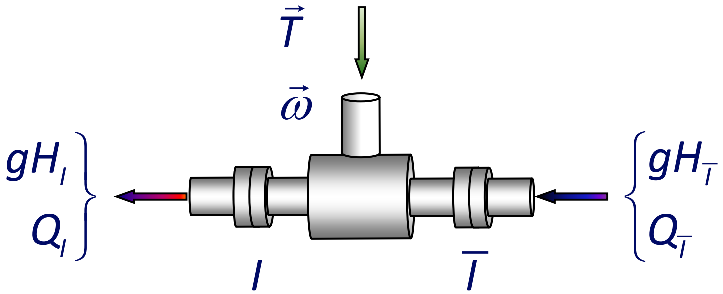
Hydraulic Characteristics: pumping mode

$$gH_I - gH_{\bar{I}} = g\left(Z_B - Z_{\bar{B}}\right) + \sum gH_r$$



Specific Hydraulic Energy

Hydraulic Brake : pumping mode



- Power input defined as negative $P = \vec{\omega} \cdot \vec{T} < 0$
- Pump Specific Energy $E \triangleq gH_I - gH_{\bar{I}} \geq 0 \quad \left(\mathrm{J \cdot kg^{-1}}\right)$
- Discharge defined as negative
- Hydraulic Power Output $P_h = \rho \times Q \times E \quad (\mathrm{W})$
- Power Input P
- Pump Efficiency $\eta^P = \dfrac{P_h}{P}$

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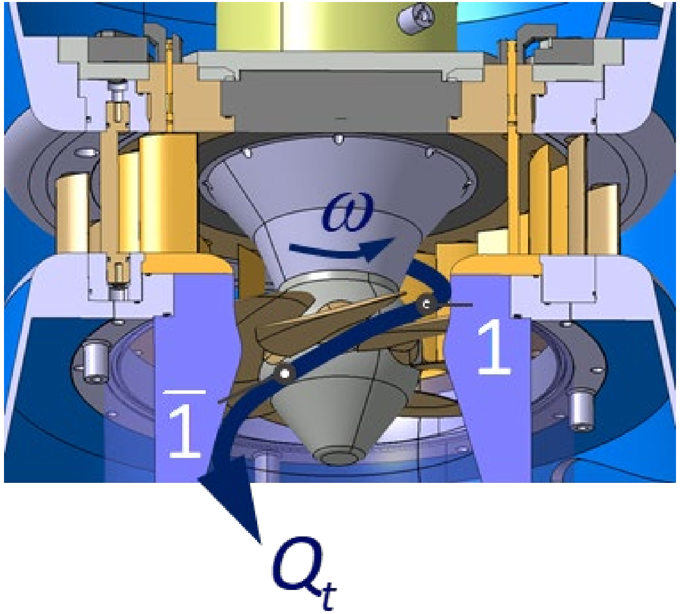
From L1: Runner/Impeller Specific energy transfer



Brillant Extension Project, British Columbia, Canada, Kaplan Turbine CAD Model, PF2 EPFL Test Rig

COLUMBIA POWER CORPORATION ALSTOM

- Traversing Discharge

  $Q_t \quad \left(\mathrm{m}^3 \cdot \mathrm{s}^{-1}\right)$

- Transferred Specific Energy

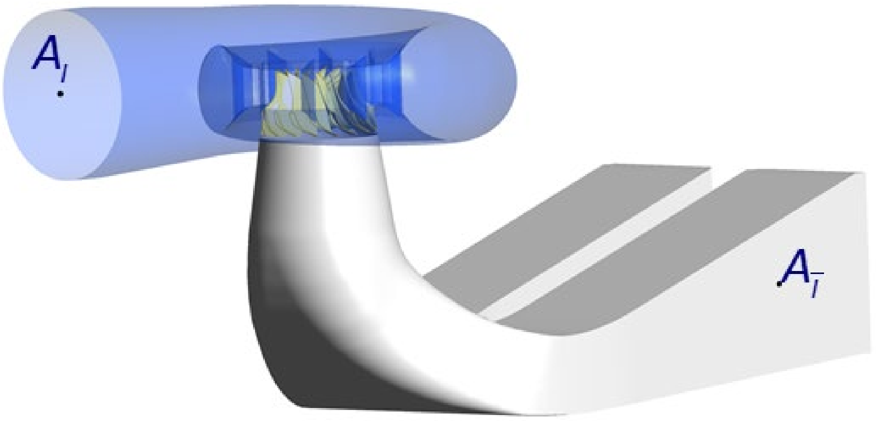
  $gH_1 - gH_{\overline{1}} = E_t \pm E_{rb} \quad \left(\mathrm{J} \cdot \mathrm{kg}^{-1}\right)$

- Power Transfer

  $P_t = \rho Q_t E_t \quad (\mathrm{W})$

  - Drive: Turbines $P > 0$
  - Brake: Pumps, Propellers $P < 0$

Power Exchange



- Hydraulic Power

$$P_h = P_I - P_{\bar{I}} \quad (\mathrm{W})$$

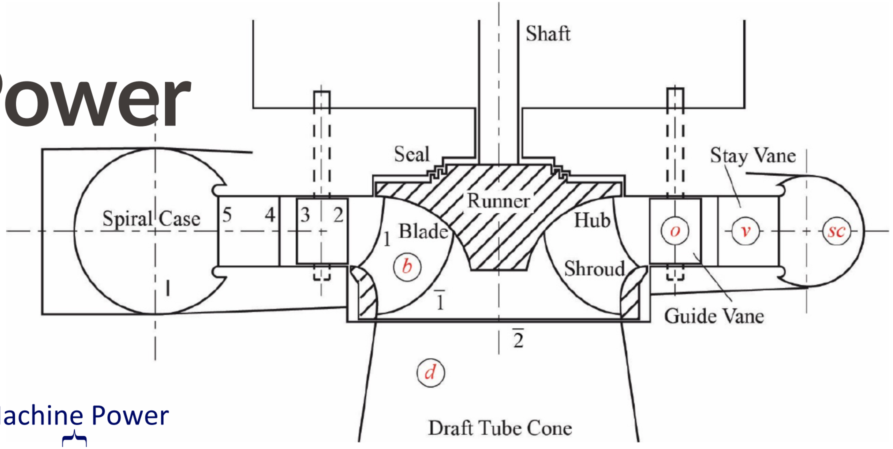
- Work-Generating:
  - Turbine

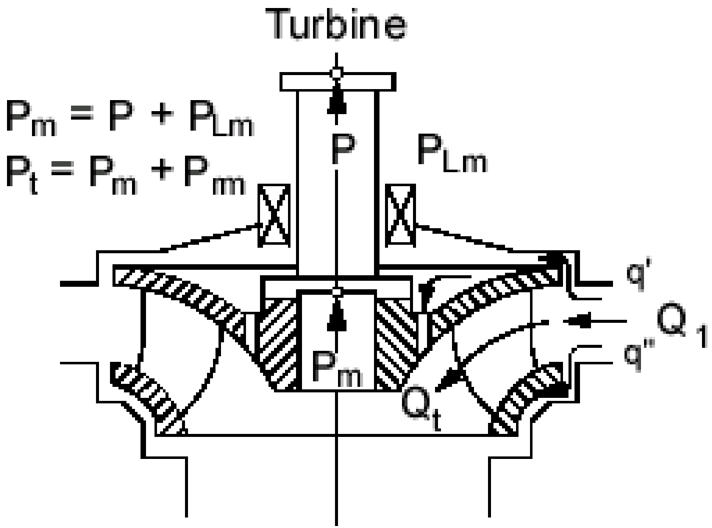
$$\eta^T P_h = P > 0$$

- Work-Absorbing
  - Pump

$$P_h = \eta^P P < 0$$

Turbine Mechanical Power





$$P_m = \overbrace{P}^{\text{Machine Power}} + \underbrace{P_{Lm}}_{\substack{\text{External Mechanical}\\ \text{Power Losses}}}$$

$$P_t = \overbrace{P_m}^{\substack{\text{Runner Mechanical}\\ \text{Power}}} + \underbrace{P_{rm}}_{\substack{\text{Internal Mechanical}\\ \text{Power Losses}}}$$

$$P_h = \overbrace{P_t}^{\text{Extracted Power}} + \underbrace{\sum P_r}_{\substack{\text{Flow Power}\\ \text{Dissipation}}} + \underbrace{\sum P_q}_{\substack{\text{Leakage Flow}\\ \text{Power}}}$$

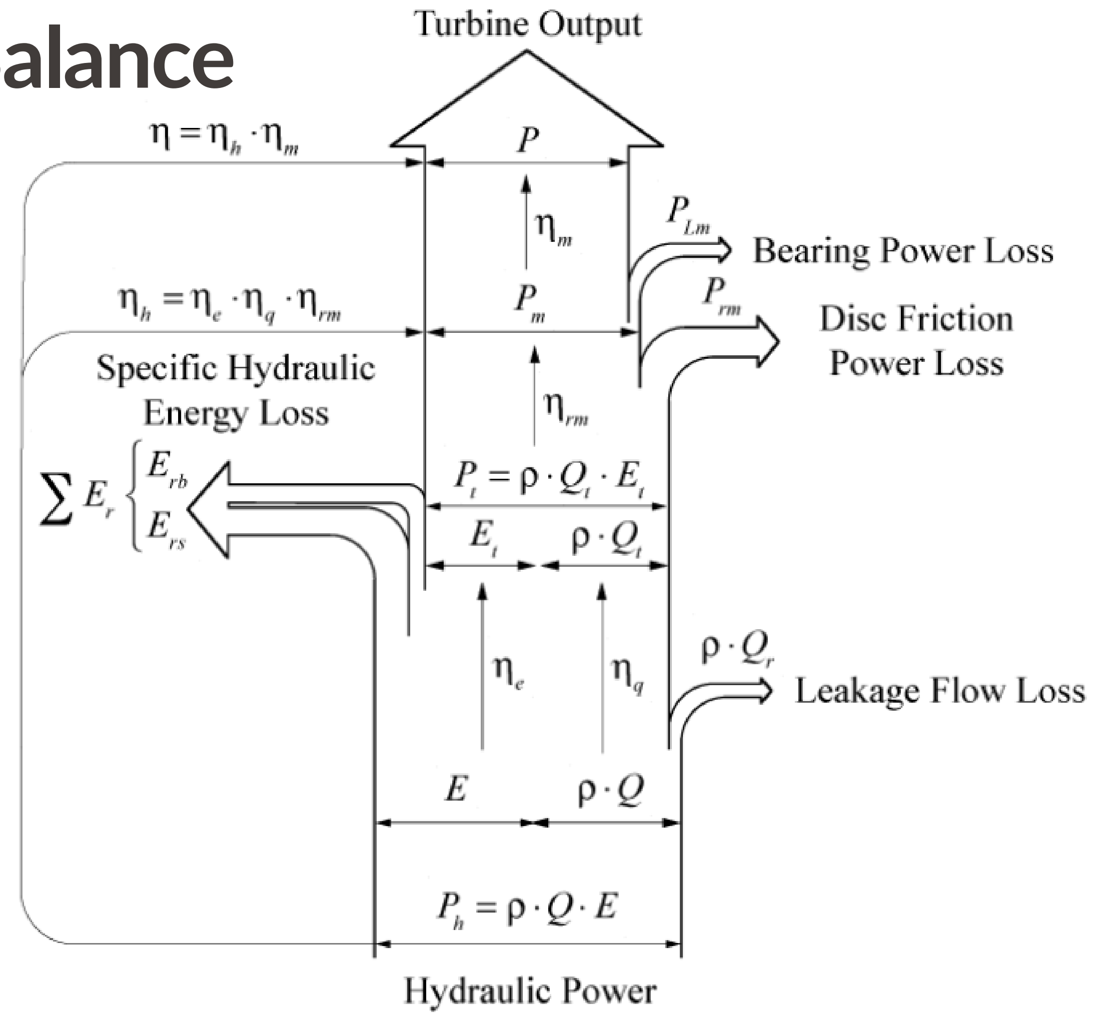
Turbine Power Balance

- Extracted Energy

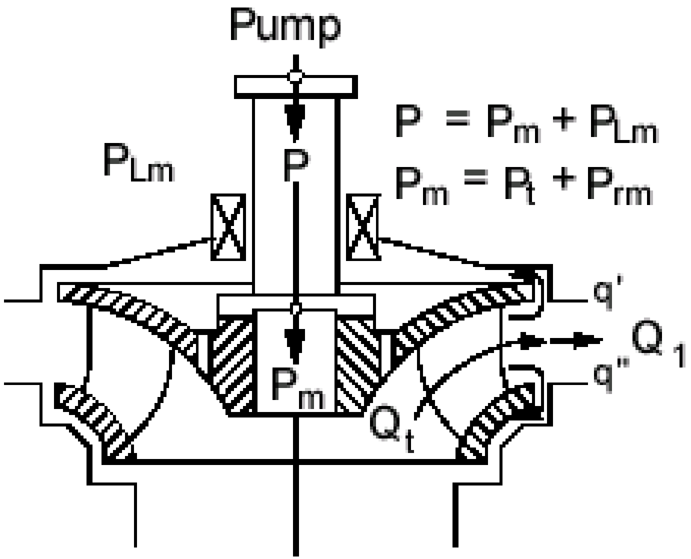
$$E_t = \frac{P_t}{\rho Q_t}$$

- Driving Torque

$$P_t = \vec{\omega} \cdot \vec{T}_t$$



Pump Mechanical Power



$$\overbrace{P}^{\text{Machine Power}} = P_m + \underbrace{P_{Lm}}_{\text{External Mechanical Power Losses}}$$

$$\overbrace{P_m}^{\text{Impeller Mechanical Power}} = P_t + \underbrace{P_{rm}}_{\text{Internal Mechanical Power Losses}}$$

$$\overbrace{P_t}^{\text{Supplied Power}} = P_h + \underbrace{\sum P_r}_{\text{Flow Power Dissipation}} + \underbrace{\sum P_q}_{\text{Leakage Flow Power}}$$

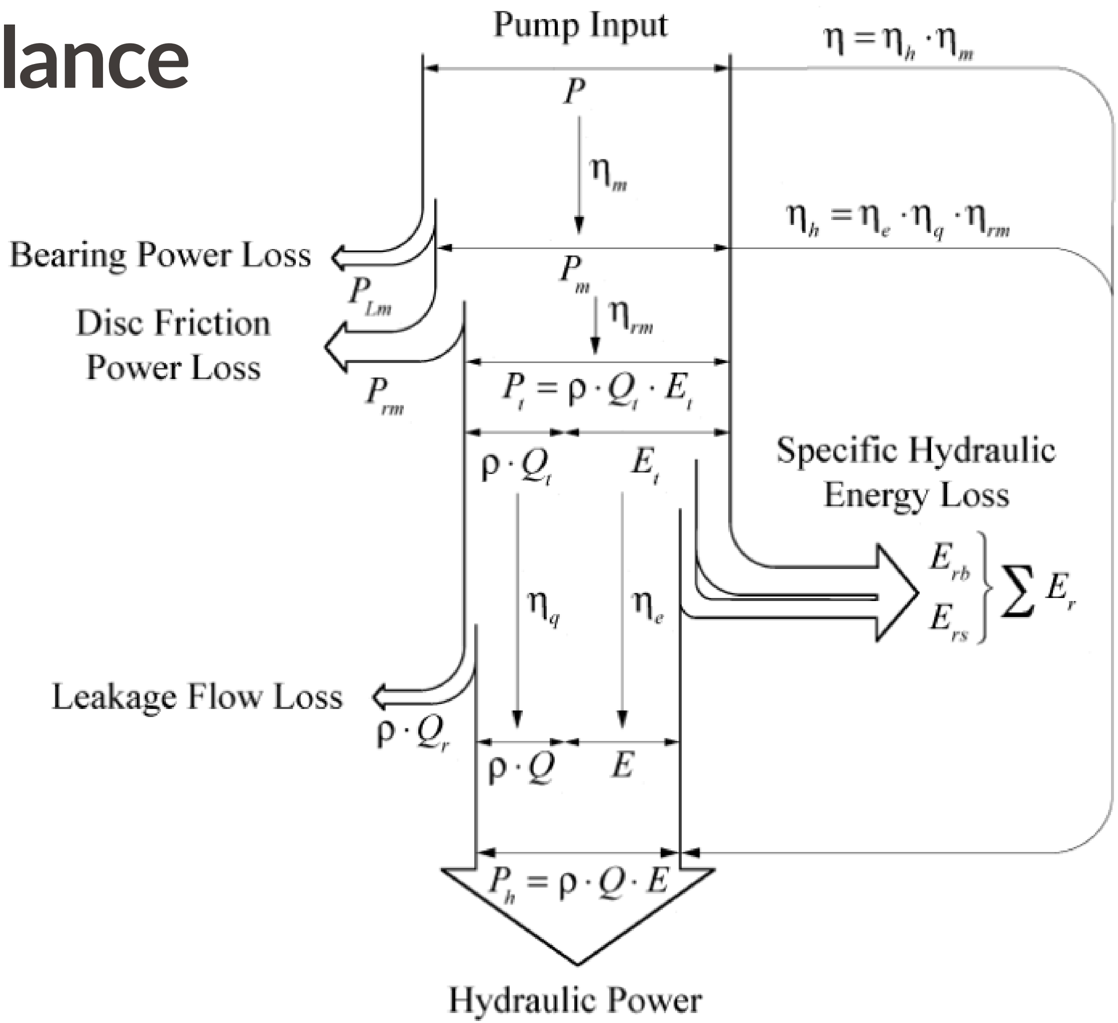
Pump Power Balance

- Supplied Energy

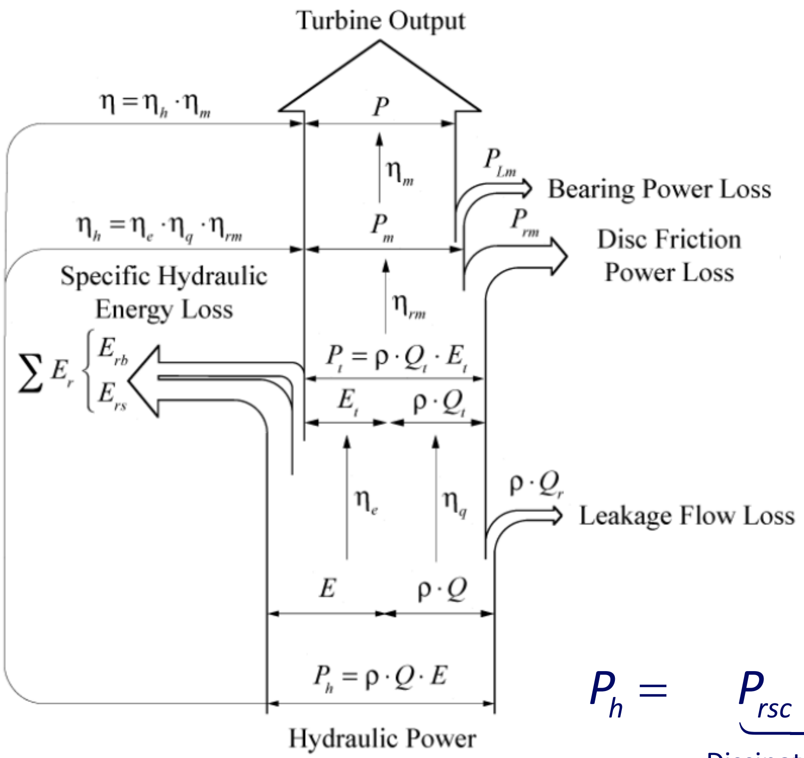
$$E_t = \frac{P_t}{\rho Q_t}$$

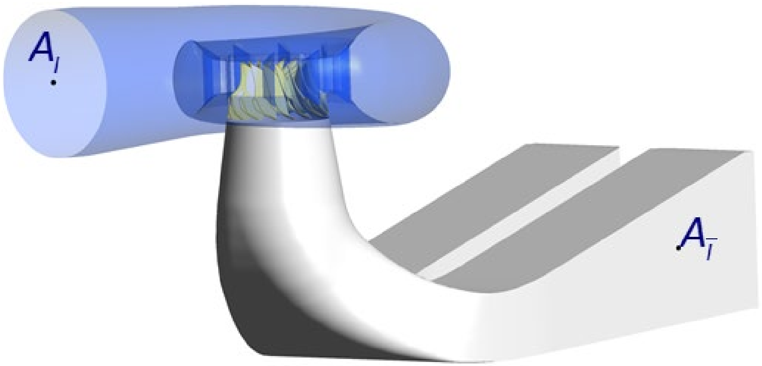
- Resisting Torque

$$P_t = \vec{\omega} \cdot \vec{T}_t$$



Power Balance





$$P_h = \underbrace{P_{rsc} + P_{rv} + P_{ro}}_{\text{Dissipation in Spiral Case, Stay and Guide vanes Cascade}} + \overbrace{P_t}^{\text{Power Transfer Runner}} + \overbrace{P_{rb}}^{\text{Dissipation in Runner}} + \overbrace{P_{rq}}^{\text{Seals Leakage}} + \underbrace{P_{rd}}_{\text{Dissipation in Draft Tube}}$$

Power Balance

Turbine Hydraulic Power Breakdown per Turbine Components

Hydraulic Power $P_h = \rho Q E$

Power lost in spiral case $P_{rsc} = \rho Q E_{rsc}$

Power lost in stay vanes $P_{rv} = \rho Q E_{rv}$

Power lost in guide vanes $P_{ro} = \rho Q E_{r0}$

Transferred Power $P_t = \rho Q_t E_t$

Power lost in the blades $P_{rb} = \rho Q_t E_{rb}$

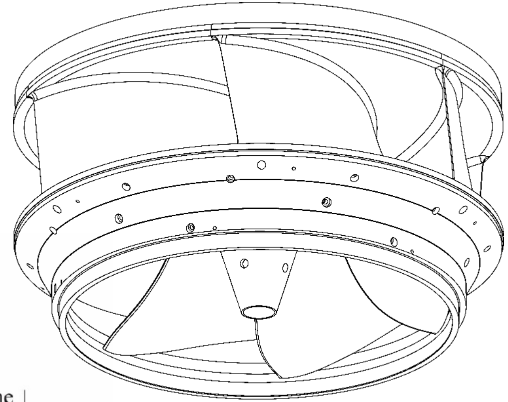
Power lost through leakage $P_{rq} = \rho q_r \underbrace{\left(E_t + E_{rb}\right)}_{=E_b = gH_1 - gH_{\bar{1}}}$

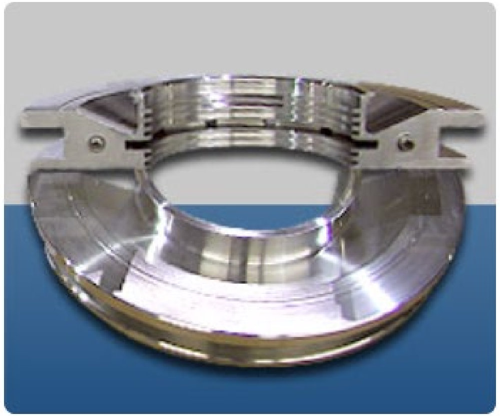
Power lost in the draft tube $P_{rd} = \rho Q E_{rd}$

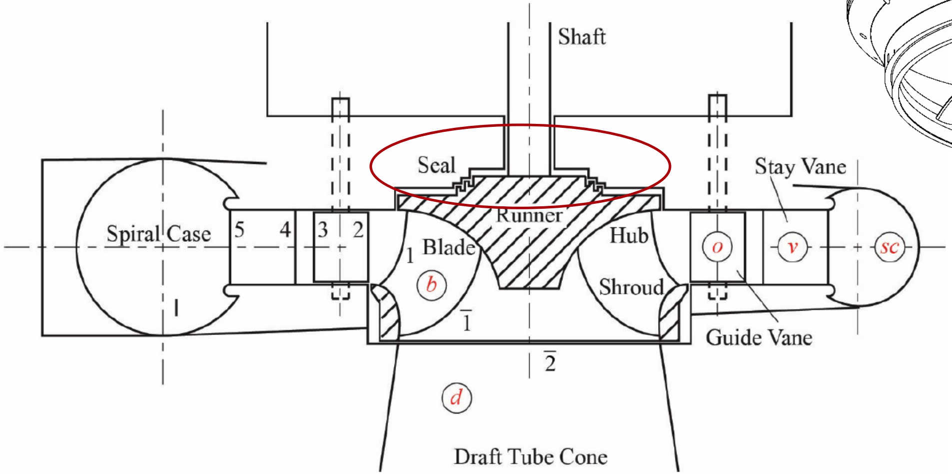
$$\left.\begin{aligned} \rho Q E = {} & \rho Q\left(E_{rsc} + E_{rv} + E_{ro}\right) \\ & + \rho Q_t E_t + \rho Q_t E_{rb} \\ & + \rho q_r \left(E_t + E_{rb}\right) \\ & + \rho Q E_{rd} \\ E = {} & E_{rsc} + E_{rv} + E_{ro} + \cancel{\frac{Q_t + q_r}{Q}}\left(E_t + E_{rb}\right) + E_{rd} \\ = {} & E_{rsc} + E_{rv} + E_{ro} + E_t + E_{rb} + E_{rd} \end{aligned}\right\}$$

HP: all leakage losses are recovered by the draft tube

Power Balance
Labyrinth Seal Leakage







- Labyrinth Energy Losses

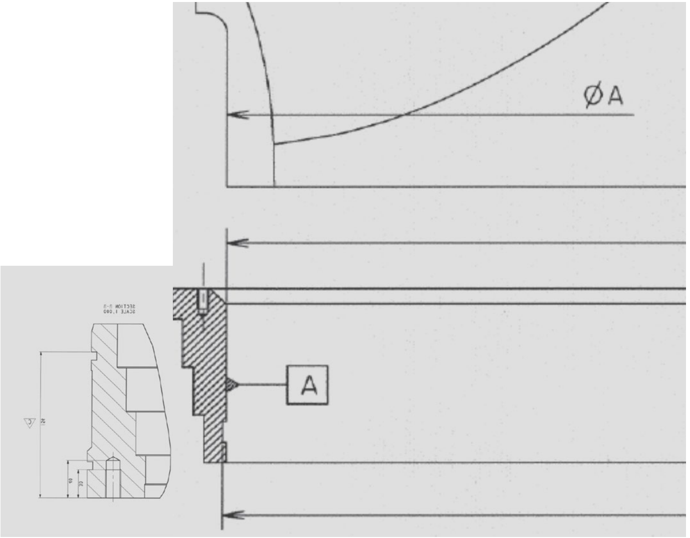
$$gH_{rSeal} = K_{VSeal} \times \frac{C_{Seal}^2}{2}$$

$$K_{VSeal} = z_{step} + \lambda\left(\mathrm{Re}, \frac{k_s}{D_{hSeal}}\right) \times \frac{\sum L_{Seal}}{D_{hSeal}}$$

$$\mathrm{Re} = \frac{C_{seal} \times D_{hseal}}{\nu}$$

Power Balance

Labyrinth Seal Leakage



- Equivalent Diameter

$$A_{Seal} = \pi\left(R_{Seal} + \delta R\right)^2 - \pi R_{Seal}^2 = \left(2 + \frac{\delta R}{R_{Seal}}\right) \times \pi R_{Seal} \times \delta R$$

$$\mathcal{P}_{seal} = 2\pi\left(R_{seal} + \delta R\right) + 2\pi R_{seal} = \left(2 + \frac{\delta R}{R_{seal}}\right) \times 2\pi R_{seal}$$

$$D_{hSeal} = \frac{4 \times A_{Seal}}{\mathcal{P}_{Seal}} = 2\delta R$$

Power Balance
Energy Efficiency

- Turbine

$$E = E_t + \Sigma E_r^T$$

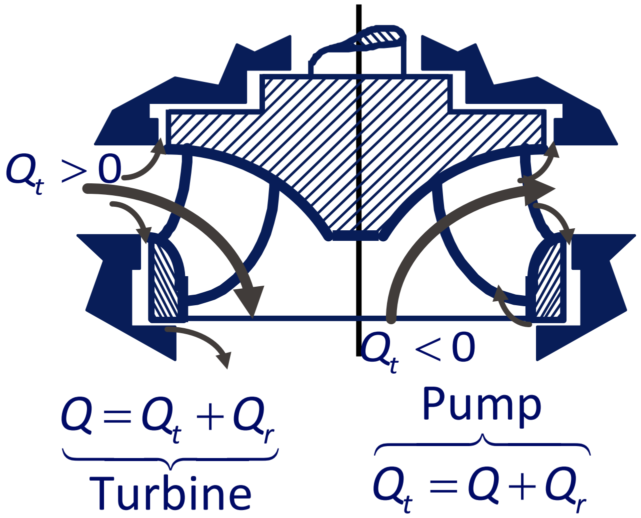
$$\eta_e^T = \frac{E_t}{E} \text{ or } 1 - \eta_e^T = \frac{\Sigma E_r^T}{E} = \Sigma e_r^T$$

- Pump

$$E_t = E + \Sigma E_r^P$$

$$\eta_e^P = \frac{E}{E_t} \text{ or } \frac{1 - \eta_e^P}{\eta_e^P} = \frac{\Sigma E_r^P}{E} = \Sigma e_r^P$$

Power Balance
Volumetric Efficiency



- Turbine

$$Q = Q_t + Q_r$$

$$\eta_q^T = \frac{Q_t}{Q} \text{ or } 1 - \eta_q^T = \frac{Q_r}{Q}$$

- Pump

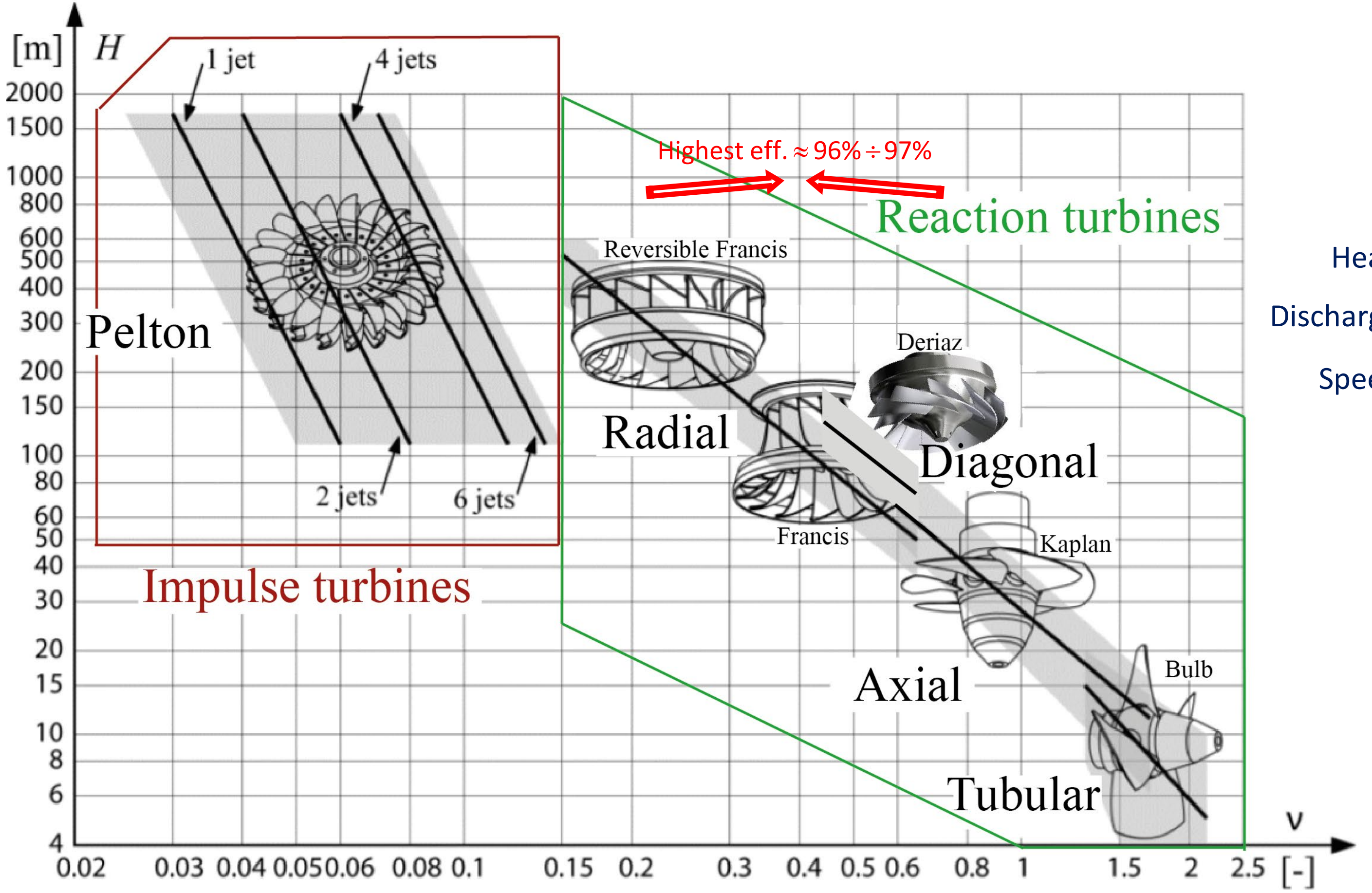
$$Q_t = Q + Q_r$$

$$\eta_q^P = \frac{Q}{Q_t} \text{ or } \frac{1 - \eta_q^P}{\eta_q^P} = \frac{Q_r}{Q}$$

Power Balance
Efficiencies definition

Efficiency definition	Transfer	Efficiency For Turbines	Efficiency For Pumps
Machine	Flow & Machinery	$\eta = \frac{P}{P_h}$	$\eta = \frac{P_h}{P}$
External Mechanical	Bearing & Ventilation	$\eta_{lm} = \frac{P}{P_m}$	$\eta_{lm} = \frac{P_m}{P}$
Internal Mechanical	Blading, Seals & Discs	$\eta_{rm} = \frac{P_m}{P_t}$	$\eta_{rm} = \frac{P_t}{P_m}$
Hydraulic	Flow, Blading, Seals & Discs	$\eta_h = \frac{P_m}{P_h}$	$\eta_h = \frac{P_h}{P_m}$

From L1: Classification of Hydraulic Runners



Head $= H$ (m)

Discharge $= Q$ $(\mathrm{m}^3 \cdot \mathrm{s}^{-1})$

Speed $= N$ (min^{-1})

Non-dimensional parameters

Specific Speed

Definition of dimensionless numbers for the specific energy conversion:

- "Power Plant" Conditions
  - Discharge $[Q] = L^3T^{-1} \cdots (\mathrm{m}^3 \cdot \mathrm{s}^{-1})$
  - Specific Energy $\left[E \triangleq gH_I - gH_{\bar{I}}\right] = L^2T^{-2} = \dfrac{ML^2T^{-2}}{M} \cdots (\mathrm{J} \cdot \mathrm{kg}^{-1})$
- Unit Characteristic
  - Angular Speed $[\omega] = T^{-1} \cdots (\mathrm{s}^{-1})$
  - Turbine/Pump Dimension $[D] = L \cdots (\mathrm{m})$

Non-dimensional parameters

Specific Speed

- Dimensionless Angular Speed Condition $\left[\nu\right]=\left[\omega\times Q^{\alpha}\times E^{\beta}\right]=M^{0}\times T^{0}\times L^{0}$

$$T^{0}\times L^{0}=T^{-1}\times L^{3\alpha}T^{-\alpha}\times L^{2\beta}T^{-2\beta}$$

- Yields to Solve the Linear System
  - Dimension of Time $-1-\alpha-2\beta=0$
  - Dimension of Length $3\alpha+2\beta=0$
- Solution $\alpha=\frac{1}{2};\quad \beta=\frac{-3}{4}$

Non-dimensional parameters
Specific Speed

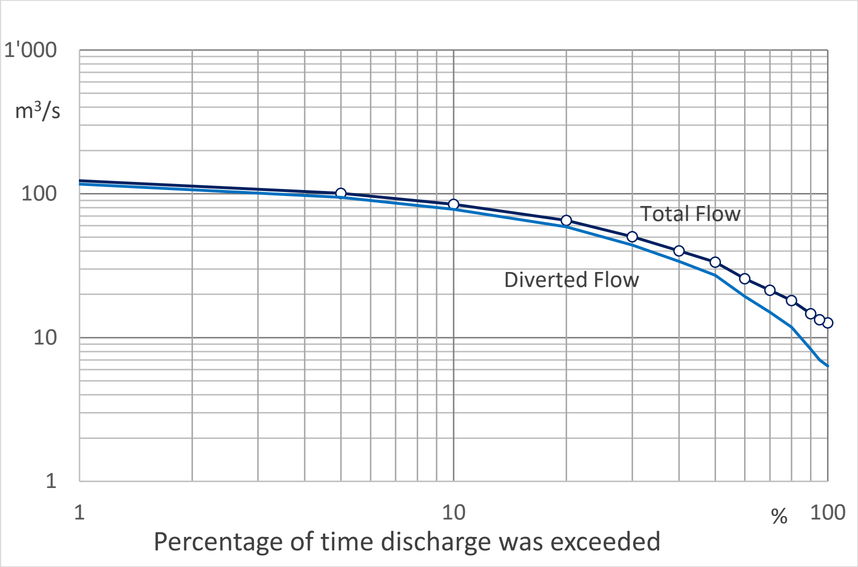
- Discharge Coefficient (Dimensionless) $\varphi = \dfrac{Cm}{U}$
- Specific Energy Coefficient (Dimensionless) $\psi = \dfrac{E}{\frac{U^2}{2}}$
- Specific Speed $\nu = \dfrac{\varphi^{\frac{1}{2}}}{\psi^{\frac{3}{4}}} = \dfrac{\omega}{\pi^{\frac{1}{2}} 2^{\frac{3}{4}}} \times \dfrac{|Q|^{\frac{1}{2}}}{E^{\frac{3}{4}}}$

Turbine Characteristic Curve

Matching Turbine Specific Speed to Site Condition

- Site Potential Specific Energy $g\left(Z_B - Z_{\overline{B}}\right)$
  - Available Specific Energy $E = gH_I - gH_{\overline{I}} = \left(g_B Z_B - g_{\overline{B}} Z_{\overline{B}}\right) - \sum gH_r \quad \left(\mathrm{J \cdot kg^{-1}}\right)$
- Site Flow Duration Statistics
  - Average Discharge

$Q_{\text{Instal.}} \quad \left(\mathrm{m^3 \cdot s^{-1}}\right)$

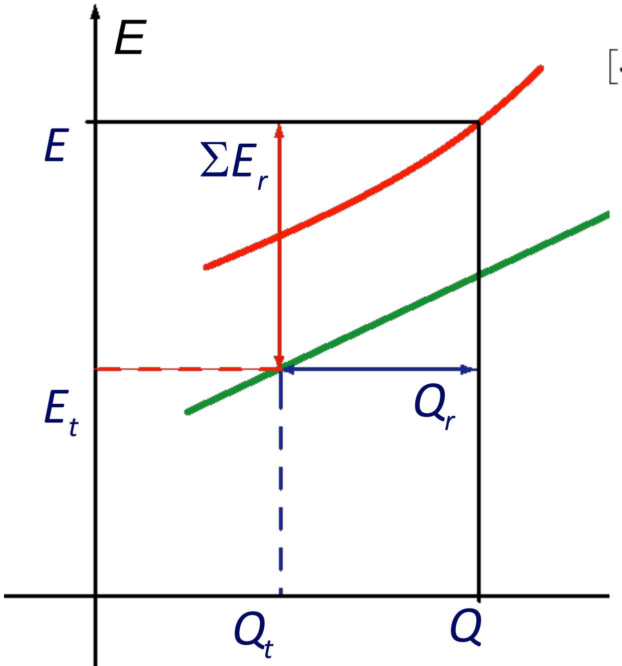


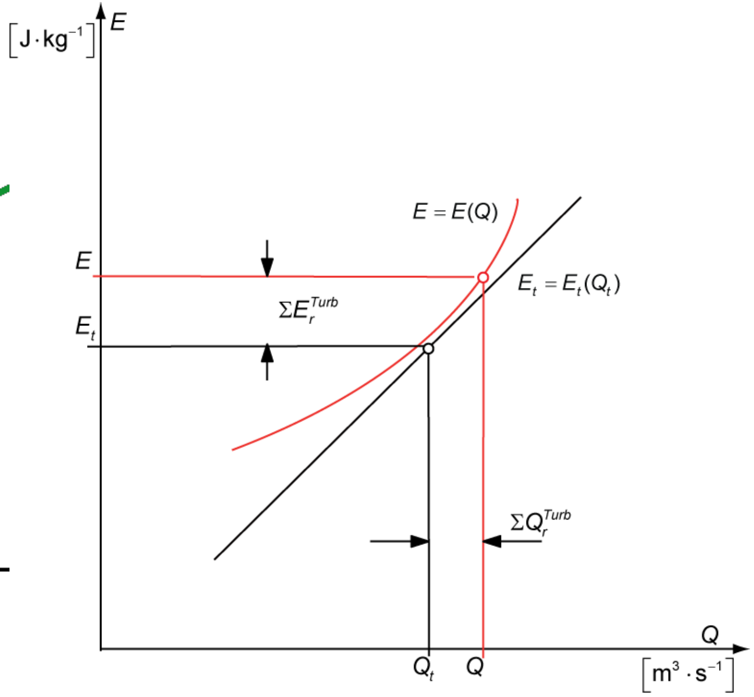
Turbine Characteristic Curve

Matching Turbine Specific Speed to Site Condition

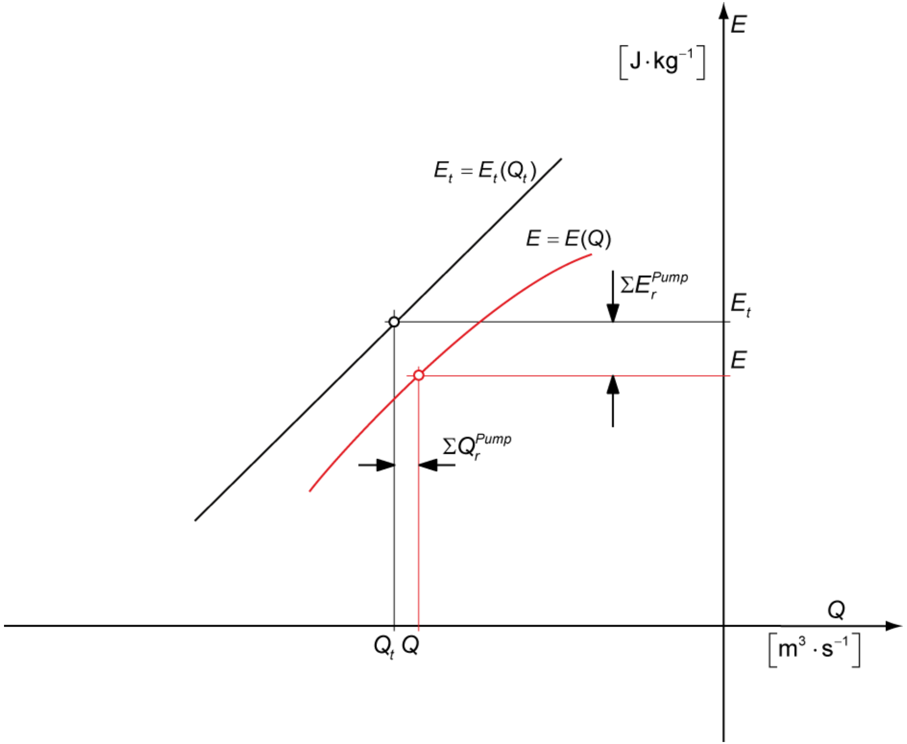
- Targeted Unit Specific Speed $\nu = \dfrac{\omega}{\pi^{\frac{1}{2}} 2^{\frac{3}{4}}} \dfrac{\left(Q_{instal.} / z_{units} \right)^{\frac{1}{2}}}{E^{\frac{3}{4}}}$
  - Rated Head $H = \dfrac{E}{g}$
  - Matching number of units z_{units}
  - Matching rotating speed $N = \dfrac{2 \times f_{grid}}{z_p} \times 60\ \mathrm{s}\ \left(\mathrm{min}^{-1}\right)$
- Check runaway speed
- Check apparent power per poles
  - Air cooling < 28 MVA < Water cooling < 35 MVA

Turbines E-Q Characteristic curve





Turbines E-Q Characteristic curve



Turbines Characteristic curve

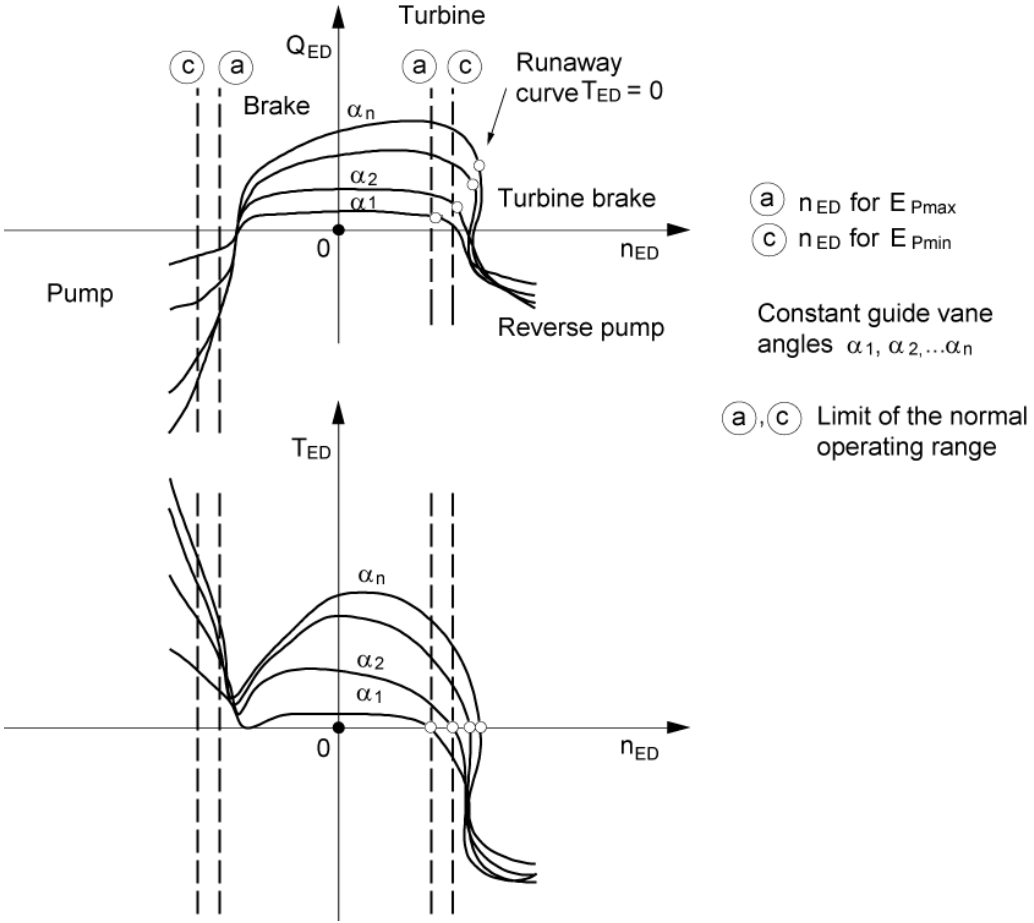
Operating range

- IEC Discharge Factor

$$Q_{ED} = \frac{Q}{D^2 E^{\frac{1}{2}}}$$

- IEC Speed Factor

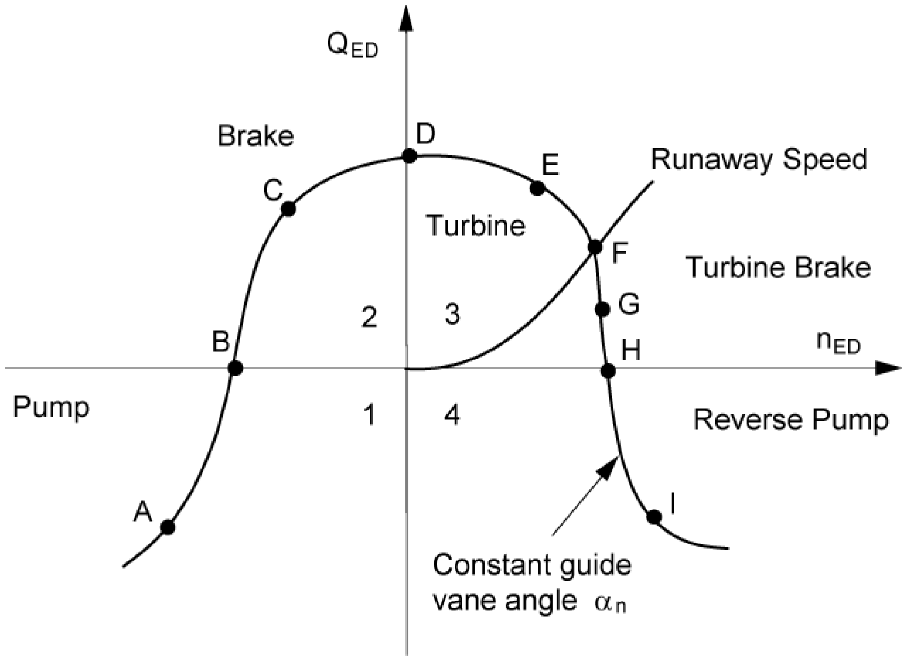
$$n_{ED} = \frac{nD}{E^{\frac{1}{2}}}$$



Classification of Hydraulic Runners

Operating range

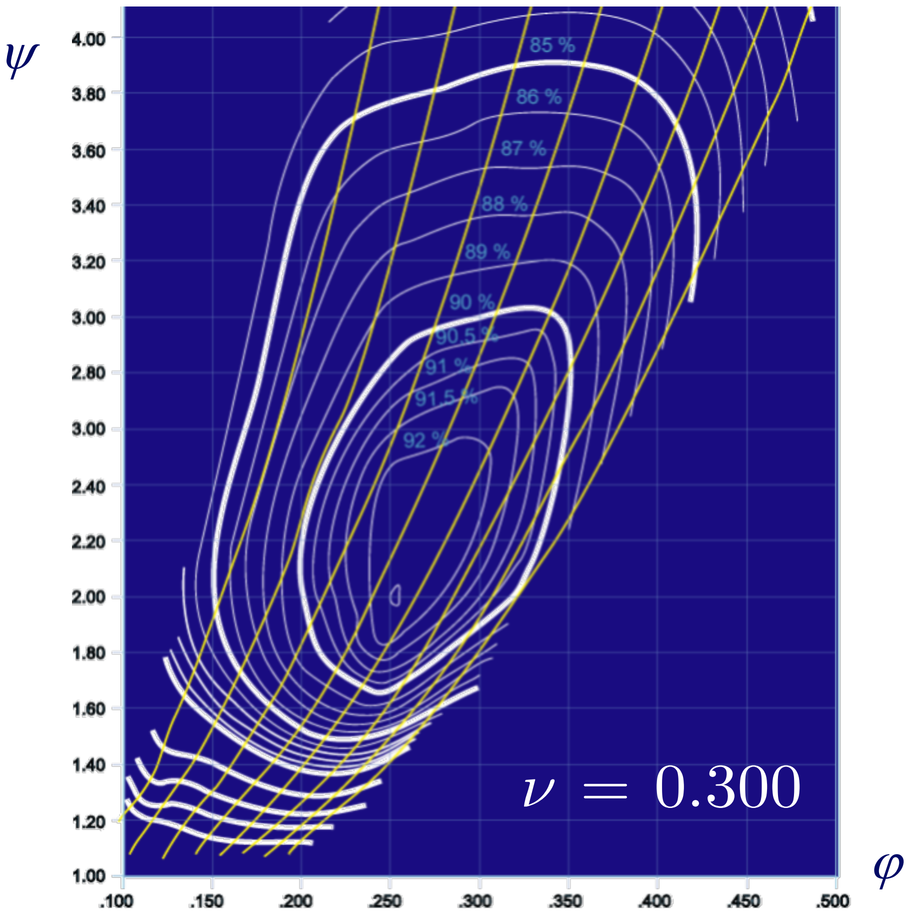
Quadrant		Direction (sign) of			Mode
Number	Name	*Q*	*n*	*T*	
1	Pump quadrant	-	-	-	Reverse turbine
				+	Pump (A)
1/2		0	-	+	Zero discharge (B)
2	Brake quadrant	+	-	+	Pump-brake (C)
2/3		+	0	+	Zero speed (D)
3	Turbine quadrant	+	+	+	Turbine (E)
				0	Runaway (F)
				-	Turbine-brake (G)
				-	Reverse rotation pump (axial machines only)
3/4		0	+	-	Zero discharge (H)
4	Reverse pump quadrant	-	+	-	Reverse rotation pump (radial machines only)
				-	Brake (I)



Machine efficiency

Efficiency hillchart

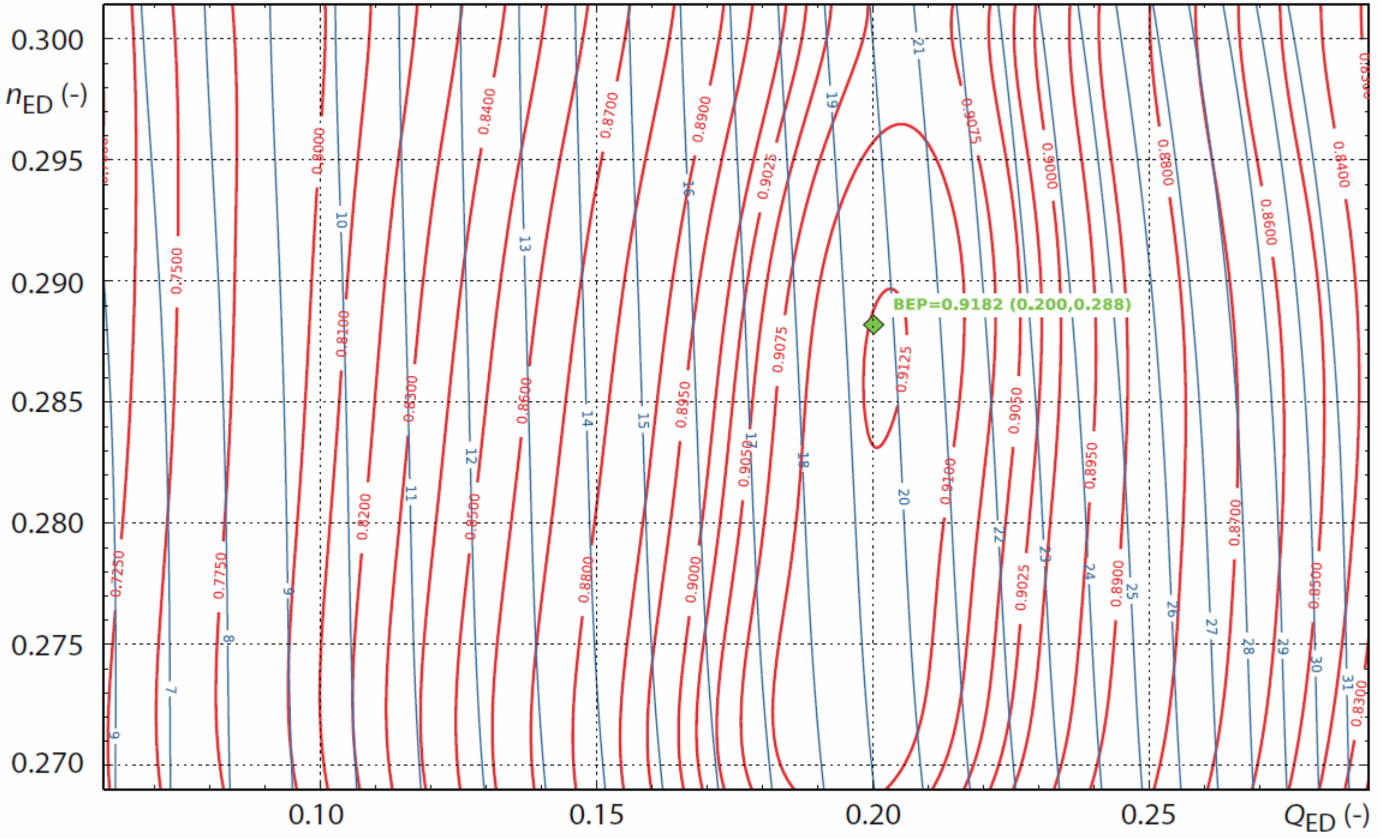
Francis turbine efficiency hillchart



EPFL

Machine efficiency

Efficiency hillchart



Machine efficiency

Efficiency hillchart – Kaplan turbine